DISCOVERING THE MAGIC OF MAGIC SQUARES

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Abstract

The purpose of this paper is to present a collection of problems that allow students to investigate magic squares and latin squares, formulate their own conjectures about these mathematical objects, look for arguments supporting or disproving their conjectures and finally establish and prove mathematical assertions. Each problem is completed with our commentary and/or experience from classrooms.

Keywords: magic square, latin square, proof, classroom activity, investigation

1 Introduction and motivation

Any arrangement of natural numbers possessing more or less expressed degree of symmetry has attracted attention since ancient time. An example of such arrangement is a *magic square*, i.e. square array consisting of natural numbers from 1 to n^2, with identical sums in all rows and columns and both diagonals.

In the mathematical classroom magic squares offer a rich source of diversified problem solving experience that range across all ability levels. The most frequent problems concerning magic squares are those in which some of numbers are already given, and the rest of them have to be found. The aim of this paper is to present such problems concerning magic squares, that a high school student may feel the flavour of scientific investigation.

We provide a collection of problems that allow students to investigate mathematical objects, formulate their own conjectures, look for arguments supporting or disproving their conjectures and finally establish and prove mathematical assertions. In order to fulfill these goals we do not start with exact definitions of investigated objects, but we stimulate students to look for important properties that finally result in commonly accepted definitions.

Our collection of problems consists of two parts. The first part deals with magic squares and their basic properties. The second part is devoted to Latin squares and their use in constructing magic squares. With each problem, we have provided additional commentary based on classroom experience. The proposed problems need very little knowledge as a prerequisite. However, our problems can stimulate students' mathematical power, which is the ability to explore, conjecture, reason logically and to use variety of mathematical methods to solve non routine problems.

We utilized this collection of problems in classrooms of 16-18 years old high school students.

2 Magic squares

Problem 2.1 Fill in the empty cells in square arrays in figure 1 by appropriate numbers. Justify your decision.

	2	3	
5	11	10	8
9	7	6	12
	14	15	

	16	9	22	
20		21		2
7	25	13	1	19
24		5		6
	4	17	10	

	29	2	4	13	
9		20	22		18
32	25	7	3	21	23
14	16	34	30	12	5
28		15	17		19
	24	33	35	8	

Figure 1: Fill in the empty cells

Classroom commentary 2.1 We suggest splitting the solution of this problem into three parts:

1. Solving the problem individually.
2. Class discussion: The students present and justify their solutions.
3. Deciding which of the proposed solutions is "the most elegant", the best justified and the most attractive from their point of view.

Usually more than half of students construct magic squares, but without consideration of the sums on diagonals. The rest of students choose other rules. While completing the second (the third) square array many of students revise the content of cell in the first (the first and the second) square because they want to apply the same rules for all three tables.

After finishing a discussion of the previous problem we propose to acquaint the students with the history of magic squares (see [1], [2] and [6]). We suggest the teacher should not formulate the definition of a magic square yet.

Problem 2.2 Construct a magic square 3×3.

Classroom commentary 2.2 Similar to the first problem we propose to split the solution process into three parts. Since we have not provided a definition of a magic square yet, students utilize only a vague description of magic square given in the historical note. Therefore they usually check only sums in rows and columns and do not consider the sums on diagonals. Moreover some of them use some numbers repeatedly or use numbers not belonging to the set $\{1, 2, \ldots, 9\}$. During the forthcoming discussion they choose as the most attractive such square arrays with 5 in the center and containing each of the numbers 1, 2, …, 9. They support their decision with the following arguments: "It is more difficult to create such square array then the others", "the number 5 is in the middle of the range 1, … , 9 and therefore it should be in the center of a square array", "such a square array has 8 identical sums, which is more than in other arrays".

We can now define *magic square of order* n as a square array $n \times n$ containing each of the numbers 1, 2, …, n^2 and having the same sums in rows, columns and on the two main diagonals. This common value is called *magic number*.

Problem 2.3 Construct a magic square of order 2.

Classroom commentary 2.3 Some students found out, that the problem has no solution, very quickly. The others examined all possible arrangements, but they hesitated to formulate the conclusion. The main goal of this problem is to show students that the answers "there is no such square", "the problem has no solution" are standard answers that may appear during solving mathematical problems.

Problem 2.4 How many magic squares of order 3 exist?

Classroom commentary 2.4 Students are usually able to find several solutions with the number 5 in the middle of the square array during the solving Problem 2.2. Many of them also notice that many of the solutions are the same ("it is the same if you turn it or flip it" - idea of the symmetry of a square). Students start to use process of trial and error to find all magic square of order 3. They usually find all 8 magic squares of order 3, but trial and error cannot prove that no other solutions exist. Students have to use logical arguments to prove that. These arguments are vague at the beginning of discussion e.g. "If 9 goes in one corner I can't complete one row or column","5 must be in the center cell", etc. In order to make their arguments more decisive the students are forced to use more exact and clear formulations.

Students together with their teacher can discuss the following solution:
The sum of the nine digits is 45. Each row and column of three adds to the same total, this total must be $45/3 = 15$. Now there are various arguments to deduce exact position of numbers. Here is one sequence of steps:
If we write number 15 as the sum of three different natural numbers, we have the following possibilities:

1+9+5	2+9+4	3+8+4	4+6+5
1+8+6	2+8+5	3+7+5	
	2+7+6		

In these sums the numbers 1, 3, 7, 9 occur 2 times, the numbers 2, 4, 6, 8 occur 3 times and the number 5 occurs 4 times. In array 3x3, the number written in arbitrary corner of array is in three triplets, such that their sum is 15. The number written in the middle of the array is in four triplets, such that their sum is 15 each. That means number 5 must be in the middle of square array and numbers 2, 4, 6, 8 in the corners of square array. Hence we observe that there are 8 possible solutions to the problem. In fact, there is only one solution and its seven reflections and rotations (see figure 2).

2	9	4
7	5	3
6	1	8

6	7	2
1	5	9
8	3	4

8	1	6
3	5	7
4	9	2

4	3	8
9	5	1
2	7	6

4	9	2
3	5	7
8	1	6

2	7	6
9	5	1
4	3	8

6	1	8
7	5	3
2	9	4

8	3	4
1	5	9
6	7	2

Figure 2: Magic square of order 3 and its seven reflections and rotations

Problem 2.5 Can two different magic squares of the same order have different magic numbers?

Classroom commentary 2.5 Very often the first reaction of students is: "Of course, it is possible." This reaction is a little bit surprising, because they determined the exact value of magic number of order 3 (in Problem 2.4) which was independent of the arrangement of numbers in magic square. In order to obtain the correct conjecture, we provide students with a few different magic squares of the same order.[1] The students have to determine the magic number of each magic square. According to new observations and obtained results, students usually formulate the correct conjecture. The correct conjecture is mostly formulated by students who found the value of the magic number in the Problem 2.4 together with more or less formal proof. We present two examples of students' arguments:

Student's proof 1: "The sum of the rows of the first magic square must be equal to its magic number multiplied by n. Similarly the sum of the rows of the second magic square is the magic number of this square multiplied by n. If the magic number corresponding to two different magic squares were different then both sums would be different. But both sums consist of the same numbers and therefore must be equal."

Student's proof 2: "I do it as in the Problem 2.4. I sum all the numbers in the magic square and then I divided the obtained result by the number of rows. The obtained

[1] As examples we can use some famous magic squares: The Diabolic magic square of order 4 founded in India in 1000, Dürer's magic square of order 4 depicted in the well-known woodcut "Melencolia I" from 1514 etc. (see e.g. [6, 2]).

sum is always the same because it does not depend on the positions of the numbers in the square array. Therefore the magic number is always the same, too."

More formally we can rewrite these proofs in the following way:

Proof 1: Let us have two magic squares of the same order n and let us denote their magic numbers by m_1 and m_2, respectively. Then, the sum of numbers of an arbitrary row of the first magic square is m_1 and the sum of all numbers in this magic square is nm_1. Analogously, the sum of the numbers in each row of the second magic square is equal to m_2 and the sum of all numbers in the second magic square is nm_2. On the other hand, $nm_1 = 1 + 2 + \cdots + n^2 = nm_2$. Therefore we have the equalities $nm_1 = nm_2$, and $m_1 = m_2$. It follows that the magic numbers of two magic squares of the same order are equal.

Proof 2: The sum of all numbers in a magic square of order n is $1 + 2 + \cdots + (n^2 - 1) + n^2 = \frac{n^2(n^2+1)}{2}$. As the sum of numbers in any row (column) of the magic square is equal to a magic number M and the number of rows (columns) is equal to n we immediately have the equality $\frac{n^2(n^2+1)}{2} = nM$. By an easy calculation we obtain that the magic number of a magic square of order n is $M = \frac{n(n^2+1)}{2}$.

Usually a teacher must help the students with technical details of the proofs. On the other hand, using the previous two proofs, a teacher can explain the differences between various proof techniques.

3 Latin and Magic Squares

Problem 3.1 Is it possible to arrange 9 saucers (3 blue, 3 yellow and 3 red) and 9 cups (3 blue, 3 yellow and 3 red) into three rows and three columns in such a way that:

1. in every row and in every column the colors of saucers are pairwise different,
2. in every row and in every column the colors of cups are pairwise different,
3. there are no two pairs of saucer and cup with the same color combination (i.e. we cannot have a blue cup on a red saucer more than once but we can have simultaneously a red cup on a blue saucer).

Classroom commentary 3.1 Students use various notations to record the solution of the problem. Some of them use pictures and/or colors, some of them use more abstract notation (see figure 3).

If no student uses number notation for recording the solution of the problem teacher can introduce a new notation (see figure 4). Students usually grasp the meaning of the notation very quickly.

Problem 3.2 Arrange 25 saucers and 25 cups (the saucers and cups are equally colored with 5 colors, say 0,. . . ,4) into 5 rows and 5 columns in such a way that:

1. in every row and in every column the colors of saucers are pairwise different,

bc bs	yc rs	rc ys
yc ys	rc bs	bc rs
rc rs	bc ys	yc bs

bc - blue cup
yc - yellow cup
rc - red cup
bs - blue saucer
ys - yellow saucer
rs - red saucer

a)

y r	b b	r y
r b	y y	b r
b y	r r	y b

cups are above
saucers are below

b)

rr	yy	bb
by	rb	yr
yb	br	ry

it is not important whether the first symbol denotes cup or saucer

c)

Figure 3: Students' notations

0 - blue
1 - yellow
2 - red

2, 2	0, 1	1, 0
0, 0	1, 2	2, 1
1, 1	2, 0	0, 2

Figure 4: Number notations

2. in every row and in every column the colors of cups are pairwise different,

3. there are no two pairs of saucer and cup with the same color combination.

Classroom commentary 3.2 On the basis of the gained experience, the problem with 25 saucers and cups can be solved relatively quickly (see figure 5). Students very often discover that they can use symmetry of a square (horizontal or vertical flip - i.e. configuration of cups and saucers differs only by a flip).

0, 4	1, 3	2, 2	3, 1	4, 0
1, 0	2, 4	3, 3	4, 2	0, 1
2, 1	3, 0	4, 4	0, 3	1, 2
3, 2	4, 1	0, 0	1, 4	2, 3
4, 3	0, 2	1, 1	2, 0	3, 4

vertical flip

0, 3	1, 4	2, 0	3, 1	4, 2
2, 1	3, 2	4, 3	0, 4	1, 0
4, 4	0, 0	1, 1	2, 2	3, 3
1, 2	2, 3	3, 4	4, 0	0, 1
3, 0	4, 1	0, 2	1, 3	2, 4

horizontal flip

Figure 5: Students' solution of Problem 3.2

Problem 3.3 Arrange 16 saucers and 16 cups (the saucers and cups are equally colored with 4 colors, say 0,. . . ,3) into 4 rows and 4 columns in such a way that:

1. in every row and in every column the colors of saucers are pairwise different,

2. in every row and in every column the colors of cups are pairwise different,

3. there are no two pairs of saucer and cup with the same color combination.

Classroom commentary 3.3 It is rather surprising for the students that the problem with 16 saucers and cups is not so easy. It takes quite a lot of time to find a solution. Examples of students' solution are in figure 6.

0, 0	2, 3	3, 1	1, 2
3, 2	1, 1	0, 3	2, 0
1, 3	3, 0	2, 2	0, 1
2, 1	0, 2	1, 0	3, 3

0, 3	1, 2	2, 1	3, 0
3, 1	2, 0	1, 3	0, 2
2, 2	3, 3	0, 0	1, 1
1, 0	0, 1	3, 2	2, 3

Figure 6: Students' solution of Problem 3.3

Now it is a good time for the teacher to tell the students something about history of the Problem 3.2 and Problem 3.3.[2]

Problem 3.4 Split the table obtained during solving Problem 3.1 into two tables. One represents the arrangement of saucers and the second represents the arrangements of cups. Do the same with the tables created in Problem 3.2 and Problem 3.3. Try to describe their properties and compare them with the properties of magic squares.

Classroom commentary 3.4 The problem provides an opportunity to introduce the concept of Latin square of order n. Students usually find out the following common properties in splitted tables (see figure 7):

2	0	1
0	1	2
1	2	0

2	1	0
0	2	1
1	0	2

0	2	3	1
3	1	0	2
1	3	2	0
2	0	1	3

0	3	1	2
2	1	3	0
3	0	2	1
1	2	0	3

0	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

4	3	2	1	0
0	4	3	2	1
1	0	4	3	2
2	1	0	4	3
3	2	1	0	4

Figure 7: Splitted tables

[2]The person who dealt with this type of problem was *Leonhard Euler* in 1779. He posed the so-called "the 36 Officer Problem": How can a delegation of six regiments, each of which sends a colonel, a lieutenant-colonel, a major, a captain, a lieutenant, and a sub-lieutenant be arranged in a regular array such that no row or column duplicates a rank or a regiment? [5]

Surprisingly, there exist no such arrangement. Euler conjectured that this problem has no solution for n^2 officers, where $n = 4k + 2$ for $k = 0, 1, 2, \ldots$. The exact solution was found in 1959. It was proved that Euler's conjecture is valid only for $k = 0, 1$. For all $k > 1$, such an arrangement exists. More information can be found in [4].

- The square array contains numbers from 0 to 2 (numbers from 0 to 3, or numbers from 0 to 4 if students work with tables created in Problem 3.3 or Problem 3.2).
- Each number in the square array occurs exactly once in each row and exactly once in each column.
- The sum of each row and column is the same.
- The sum of each row and column is equal to 3 (to 6, or to 10).

Now we define *Latin square of order n* as a square array $n \times n$ containing each of the numbers $0, 1, \ldots, n-1$ in such a way, that each number occurs exactly once in each row and exactly once in each column (see figure 8).

0	1	2	...	$n-1$	n
1	2	3	...	n	0
⋮	⋮	⋮	...	⋮	⋮
$n-1$	n	0	...	$n-3$	$n-2$
n	0	1	...	$n-2$	$n-1$

Figure 8: A Latin square of order n

We introduce the following operations with Latin squares:

- Addition of a constant a; every element x of a Latin square is replaced with the number $a + x$, (see figure 9)

$$5 + \begin{array}{|c|c|c|}\hline 0 & 2 & 1 \\ \hline 2 & 1 & 0 \\ \hline 1 & 0 & 2 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 5 & 7 & 6 \\ \hline 7 & 6 & 5 \\ \hline 6 & 5 & 7 \\ \hline \end{array}$$

Figure 9: Addition of a constant

- Multiplication by a constant a; every element x of a Latin square is replaced with the number $a \times x$, (see figure 10)

$$4 \times \begin{array}{|c|c|c|c|}\hline 0 & 1 & 2 & 3 \\ \hline 2 & 3 & 0 & 1 \\ \hline 3 & 2 & 1 & 0 \\ \hline 1 & 0 & 3 & 2 \\ \hline \end{array} = \begin{array}{|c|c|c|c|}\hline 0 & 4 & 8 & 12 \\ \hline 8 & 12 & 0 & 4 \\ \hline 12 & 8 & 4 & 0 \\ \hline 4 & 0 & 12 & 8 \\ \hline \end{array}$$

Figure 10: Multiplication by a constant

- The sum of two Latin squares of the same order (or square arrays, which were created from Latin squares in terms of the operations introduced above), (see figure 11); each cell of the resulting square array contains the sum of the elements of the corresponding cells of the summands

0	1	2	3
2	3	0	1
3	2	1	0
1	0	3	2

+

0	3	1	2
2	1	3	0
3	0	2	1
1	2	0	3

=

0	4	3	5
4	4	3	1
6	2	3	1
2	2	3	5

Figure 11: Sum of square arrays

Problem 3.5 Take square arrays obtained from the solution of Problem 3.4. Apply the operations described above to the obtained square arrays (one example is in figure 12) and investigate the properties of the resulting tables. What can be said about the sums of the numbers in the rows and columns of the new tables?

$2\ +$

0	2	1
2	1	0
1	0	2

$+\ 4\ \times$

1	2	0
0	1	2
2	0	1

Figure 12: Combining operations

Classroom commentary 3.5 One can easily observe that an addition of a constant and a multiplication by a constant preserve constant sum in the rows and columns. On the other hand, it is not so obvious that a combination of all three introduced operations preserves row and column sums. Therefore we can ask students to try to find a counterexample. Usually some students claim that even the repeated applications of these operations (in an arbitrary order) leads to a square array that has the same sum in all its rows and columns. Hence we can ask them to prove this assertion. The proof can look as follows: "If we add a constant c to a Latin square of order n, then the sums in each row (column) increase by nc and therefore they remain equal to each other. Similarly, if we multiple a Latin square of order n by a constant c then the sum in each row (column) increases c times and the claim is again valid. When we sum two square arrays, one with row/column sums equal to a and the second one with the row/column sums equal to b, then by an easy application of associative law we obtain that the row/column sums in the resulting square array are equal to $a+b$. Finally, if we combine all three operation, then by repeated applications of the arguments mentioned above we get that the sums in the all rows and column of the resulting square array have the same value."

Problem 3.6 Apply the described operations with the pair of Latin squares of order 3 in such way that

a) the biggest number in the resulting table is 9,

b) all the numbers in the resulting table are pairwise different,

c) all the numbers in the resulting table are pairwise different and the biggest number in the table is 9.

Classroom commentary 3.6 Students usually add the number 7 to a Latin square of order 3 and they get the right table in part a). Other frequent solution is multiplying a Latin square of order 3 by the number 4 and adding 1. Part b) is more difficult. The following assertion appeared frequently: "We need to multiply one square array with quite big number to get right solution, I tried it with number 100."(see figure 13)

$$100 \times \begin{array}{|c|c|c|}\hline 0 & 1 & 2 \\ \hline 1 & 2 & 0 \\ \hline 2 & 0 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline 0 & 2 & 1 \\ \hline 1 & 0 & 2 \\ \hline 2 & 1 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 0 & 102 & 201 \\ \hline 101 & 200 & 2 \\ \hline 202 & 1 & 100 \\ \hline \end{array}$$

Figure 13: A student's solution of Problem 3.6 b)

I asked them the question: "Can we use a smaller number?". Then we got a solution using the number 5 (see figure 14) and later the number 3 (see figure 15). We also showed that for number 2 we cannot obtain different numbers in the resulting table.

$$5 \times \begin{array}{|c|c|c|}\hline 0 & 1 & 2 \\ \hline 1 & 2 & 0 \\ \hline 2 & 0 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline 0 & 2 & 1 \\ \hline 1 & 0 & 2 \\ \hline 2 & 1 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 0 & 7 & 11 \\ \hline 6 & 10 & 2 \\ \hline 12 & 1 & 5 \\ \hline \end{array}$$

Figure 14: 2nd student's solution of Problem 3.6 b)

$$3 \times \begin{array}{|c|c|c|}\hline 0 & 1 & 2 \\ \hline 1 & 2 & 0 \\ \hline 2 & 0 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline 0 & 2 & 1 \\ \hline 1 & 0 & 2 \\ \hline 2 & 1 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 0 & 5 & 7 \\ \hline 4 & 6 & 2 \\ \hline 8 & 1 & 3 \\ \hline \end{array}$$

Figure 15: 3rd student's solution of Problem 3.6 b)

Students solve the part c) using the resulting table in figure 15 and adding the number 1 to the table.

Problem 3.7 Produce a magic square of order 3 from two Latin squares created in Problem 3.4 by the described operations. Can you create a magic square from arbitrary two Latin squares?

Classroom commentary 3.7 The solution of the problem should result in the following finding: If we want to get a magic square, generating Latin squares must have the property that not only the sum of each row and column is the same, but also the sum of two diagonals is the same. For example, students do not obtain a magic square of order 3 in figure 16 because the two diagonal sums are not equal to 3 (one is equal to 6 and the other is equal to 0). In figure 17 each diagonal sum is equal to 3 and students get a magic square.

Problem 3.8 Consider pairs of Latin squares of order 4 that were created in Problem 3.4. Apply the described operations to these squares in such a way that

$$3 \times \begin{array}{|c|c|c|}\hline 0 & 1 & 2 \\ \hline 1 & 2 & 0 \\ \hline 2 & 0 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline 0 & 2 & 1 \\ \hline 1 & 0 & 2 \\ \hline 2 & 1 & 0 \\ \hline \end{array} + 1 = \begin{array}{|c|c|c|}\hline 1 & 6 & 8 \\ \hline 5 & 7 & 3 \\ \hline 9 & 2 & 4 \\ \hline \end{array}$$

Figure 16: A square array that is not a magic square

$$3 \times \begin{array}{|c|c|c|}\hline 1 & 0 & 2 \\ \hline 2 & 1 & 0 \\ \hline 0 & 2 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline 2 & 0 & 1 \\ \hline 0 & 1 & 2 \\ \hline 1 & 2 & 0 \\ \hline \end{array} + 1 = \begin{array}{|c|c|c|}\hline 6 & 1 & 8 \\ \hline 7 & 5 & 3 \\ \hline 2 & 9 & 4 \\ \hline \end{array}$$

Figure 17: Magic square of order 3

a) the biggest number in the resulting table of order 4 is 16,

b) all the numbers in the resulting table are pairwise different,

c) all the numbers in the resulting table are pairwise different and the biggest number in the table of order 4 is 16,

d) the resulting table is a magic square of order 4.

Classroom commentary 3.8 This problem students solve individually. Sometimes they have to modify their Latin squares in order to be able to construct a magic square (sum on each diagonal must be equal to 6). A solution for magic square of order 4 is depicted in figure 18.

$$4 \times \begin{array}{|c|c|c|c|}\hline 0 & 1 & 2 & 3 \\ \hline 2 & 3 & 0 & 1 \\ \hline 3 & 2 & 1 & 0 \\ \hline 1 & 0 & 3 & 2 \\ \hline \end{array} + \begin{array}{|c|c|c|c|}\hline 0 & 3 & 1 & 2 \\ \hline 2 & 1 & 3 & 0 \\ \hline 3 & 0 & 2 & 1 \\ \hline 1 & 2 & 0 & 3 \\ \hline \end{array} + 1 = \begin{array}{|c|c|c|c|}\hline 1 & 8 & 10 & 15 \\ \hline 11 & 14 & 4 & 5 \\ \hline 16 & 9 & 7 & 2 \\ \hline 6 & 3 & 13 & 12 \\ \hline \end{array}$$

Figure 18: Magic square of order 4

Problem 3.9 Consider pairs of Latin squares of order 5 that were created in Problem 3.4. Apply the described operations to these squares in such a way that the resulting table is a magic square of order 5.

Classroom commentary 3.9 As in the previous problem, students usually had to modify their Latin square in order to be able to construct magic square.

4 Conclusion

Mathematics classroom should be a place where students learn to value mathematics, to communicate their ideas, become confident in their own mathematical abilities, learn to make, refine and explore conjectures and use a variety of reasoning to confirm or disprove those conjectures. All students can feel the thrill of success through exploration and hands-on experimentation [3]. The activities concerning magic squares are an interesting outlet for achieving these goals. Our experiences have shown that the problems presented in this paper challenge high school students while reinforcing their "mathematical power".

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